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Why does the meter beat the second?

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Abstract

The French Academy units of time and length — the second and the meter — are traditionally considered independent from each other. However, it is a matter of fact that a meter simple pendulum beats virtually the second in each oscillation, a 'surprising' coincidence that has no physical reason. We shortly review the steps that led to the choice of the meter as the ten millionth part of the quadrant of the meridian, and rise the suspicion that, indeed, the length of the seconds pendulum was in fact the starting point in establishing the actual length of the meter.

1 Introduction

Presently, the unit of length in the International System (SI) [1] is a derived unit, related to the unit of time via the speed of light, assumed to be constant and of exact value $c = 299\,792\,458\,\mathrm{m/s}$.[1] The unit of time is related to the period of a well defined electromagnetic wave.[2] The proportionality factors that relate conventional meter and second to more fundamental and reproducible quantities were chosen to make these units back compatible to the original units of the Metric System. In that system the second was defined as 1/86400 of the average solar day and the meter was related to the Earth meridian (namely 1/10 000 000 of the distance between the pole and the equator on the Earth's surface). Therefore, meter and second were initially not formally related to each other.

It is then curious to discover — we say 'discover' because this simple observation has surprised us, as well as many colleagues and friends — that a simple pendulum of one meter in length 'beats' the second, in the sense that each oscillation, i.e. half period, takes approximately one second.

[1] A meter is the distance covered by light in vacuum in 1/299 792 458 of a second.

[2] A second is equal to the duration of 9 192 631 770 periods of the radiation corresponding to the transition between two hyperfine levels (F=4, M=0 and F=3, M=0 of the fundamental status $^2S_{1/2}$) of the atom Cesium 133.

A simple exercise (see Appendix A) shows that there is no deep physical connection between the two quantities. Other explanations are a mere, indeed remarkable numerical coincidence, or the fact that the 'first meter' was already somehow a derived unit — although never explicitly stated in official documents — in the sense that particular fraction of that particular geometrical parameter of Earth was chosen to approximately match the length of the pendulum that beats the second.

In order to understand how believable the guess of an initially derived meter is, we have gone through the steps that led to the several proposals of a unit of length taken from nature. We have learned an interesting circular story of a unit of length that begins and ends with the second. At the beginning the unit of length was bound to the second by gravitational acceleration, taken at a reference point on the Earth's surface. Now the meter is bound to the second by the speed of light. In between (and somehow in parallel, at the very beginning) there have been the several attempts to define *"a unit of length that does not depend on any other quantity"*[3] [2]: initially from the size of Earth; then from a conventional platinum bar; finally from the wavelength of a well defined electromagnetic radiation.

The recent relationship between the meter and the second is quite well known. Much less known is the fact that, just before the French Academy definition of the meter as a fraction of the Earth meridian, most scientists had considered the length of the pendulum that beats the second to be the most suitable unit of length. That consensus had steadily increased during the 18th century. However, in spring 1791 there was a metrological revolution inside the French Revolution. The result was a unit of length equal to a quadrant of the Earth's meridian (with practical unit a convenient decimal sub-multiple, namely 1/10 000 000).

Our interdisciplinary work wants to reason the plausibility of a meter initially bound to the second.

2 From anthropomorphic units to units taken from nature

It is universally accepted that the first important stage in the development of metrological concepts related to measures of length is the anthropomorphic one, in which the main units of measurement are the parts of the human body [3, 4]. As the sociologist and historian of metrology Witold Kula puts it, *"man measures the world with himself"* [4] — a variation of Protagoras' *"man is the measure of all things"*. It is a very ancient and primitive approach. Certainly, even the first people who adopted such units must have been aware that the length of their own feet or fingers was different from their neighbor's ones. But initially such personal differences did not seem important, given the low degree of accuracy required in measurements in that social context.

Later the anthropomorphic approach reached a first level of abstraction, characterized by *"the shift from concrete representations to abstract ones, from 'my or your finger' to 'finger in general' "*[4] [4]. Nevertheless, even when the stage was reached of conceiving measurement units as abstract concepts, differences in establishing the value of these units remained, depending on region or time [6, 7, 8, 9].

Only in the eighteenth century, with the consolidation of the experimental method on one hand, and the drive towards international co-operation and trading on the other,

[3] All English quotes not referring to English bibliography are translation by the authors.

[4] The earliest measurement standard we have evidence of is the *Egyptian cubit*, the length of the forearm from elbow to fingers, realized around 2500 B.C. in a piece of marble of about 50 centimeters [5].

strong emphasis was placed for the first time on the need for standardized units. In fact, the plethora of units in use in the various countries, and even in different towns and regions of the same country, made it difficult for researchers at different places, observing the same physical phenomenon, to quantitatively compare their results. In addition, the coveted improvement in international commerce was in this situation strongly hampered.

Changing a system of weights and measures (and currency) in a country encounters notoriously much resistance, due to reasons of several natures: *practical*, like having to renew all instruments of measurement and converting values between old and new systems in order to be back compatible with all the information (measurements, book keeping and contracts) acquired in the old system; *sociological*, due to human tendency to make profit of somebody else's ignorance or good faith (the recent switch from national European currencies to Euro is a good example of it, especially in the authors' country); *psychological*, that, apart from a general inertia to changes of a large fraction of people, is objectively due to the fact that adults have difficulty in modifying their mental representations (and, taking again the Euro introduction as an example, we authors still use to roughly convert Euro values into 'thousand Lire' — the practical unit of currency in Italy before the Euro — multiplying the Euro value by two). National laws can help to impose the new system,[5] but the process of changing can take decades and sometimes it can simply result into a fiasco.[6]

The problem becomes even more difficult to solve when one aims to reach an international standardization of units. To the above mentioned problems, we have to add *national pride* that refuses to accept foreign rules, no matter how reasonable they might be. Indeed, a similar regional pride existed in the eighteenth century also among towns and regions of the same country. Any attempt of standardization would have then been perceived as an imposition of the most powerful town or region over the others.[7]

[5] With this respect, changes of currencies are easily ruled by central banks, especially in modern times in which banknotes and coins have no intrinsic value.

[6] In history there are plenty of examples of this kind, like the difficulty of France to establish the decimal metric system or the failed Carolingian and Renaissance reforms [9]. Just to make a recent and practical example, Italy has adopted the International System (SI) since the mid seventies, and other units were banned by law. Nevertheless, after thirty years, though the SI unit of power is the Watt, car power is quoted in HP (yes, official documents do have kW, but drivers, sellers and media only speak of HP), centralized home heating power in kCal/h, (but small electric heaters are given in Watt) and air conditioning cooling power in Btu/h. What is bad is that, contrary to centimeters and inches, where people know that the units measure the same thing but citizens of different countries have a mental representation in either unit, the average Italian does not even know that all the above units measure the same thing and practically none has a mental representation of a Btu/h, arrived to us in mass with air conditioners in the last few years. Therefore, people don't even suspect that it is possible to convert Btu/h to Watt to have a better perception of what 7000 Btu/h might mean (a cooling power of about 2 kW).

[7] For example, this was one of the possibilities envisaged in France before the metric system: to extend to all France the system used in Paris. This was still the proposal of Joseph-Jérôme Lalande in April 1789. And he strenuously defended it later against the meter. Though his figure is often presented as conservative, for his opposition to the meter, we have to admit that Lalande was quite right in proposing to base the unit of length on a physical standard, like the Paris *toise*, rather than on the size of Earth. This is exactly what happened one century later, when in 1889, having metrologists realized that a practical unit based on Earth was not accurately reproducible as required for precision measurements and, most important, the definition itself was basically flawed, as we shall see at the end of this paper. The definition of the unit of length was then solely based upon the platinum standard, with no reference to Earth any more. Perhaps what Lalande underestimated was the psychological driving force of standards taken from nature that, with all the problems he correctly spotted (see

This is the reason that led several scientists and thinkers to seek for units that were not just as arbitrary as any materialization of the parts of the human body. Standards 'taken from nature' (*"pris dans la nature"* [2]) were then seen as a possible chance to achieve a reform of units of measurements acceptable to all citizens of the country and possibly to all nations. Besides these sociological aspects, well chosen units based on nature have advantage of being constant with time and reproducible in the case that the practical standards are corrupted or destroyed.[8]

The problem of having a universal system of units of measurement was first tackled seriously about at the same time in France, Great Britain and United States at the end of the 18th century, though the only program eventually accomplished was that launched by the French National Assembly at the time of the French Revolution.

The French program started at the beginning of 1790 when the Academy was entrusted for the reform by the National Assembly. As the historian Alder puts it [10], one of the aims of the French scientist was to facilitate communications among scientists, engineers, and administrators: their *"grander ambition was to transform France - and ultimately, the all world - into a free market for the open exchange of goods and information"*. (For some 'external' influences — economical, philosophical, political and cultural — on the works of the French Academy see e.g. Refs. [9, 12, 13].) However, for the French legislator the issue had primarily a sociological priority, as can be perceived in the project of the 1793 decree [14] in which *citoyen* Louis Arbogast requires the attention of the National Assembly on a "*subject of universal beneficence*, [having been] *the uniformity of weights and measures for a long time one of the philanthropist wishes; claimed at once by the sciences and the arts, by commerce and by the useful man who lives of the work of his hands and who, the most exposed to frauds, is the less capable of bearing their effects.*"

The success of the French attempt to create a universal system of units of measurement was due to several reasons that, besides the driving illuministic spirit, certainly include the political drive to support the project with education and imposition, even when the revolution had ended. But perhaps the main reason that forced rulers to be so determined was the special chaotic situation of units of measurement there at that time [15], where about 800 different names for measures have been estimated, whose units varied in different towns, for a total of about 250000 differently sized units [16]. *"It is quite evident that the diversity of weights and measures of different countries, and frequently in the same province, are a source of embarrassment in commerce, in the study of physics, in history, and even in politics itself; the unknown names of foreign measures, the laziness or difficulty in relating them to our own give rise to confusion in our ideas and leave us in ignorance of facts which could be useful to us"*, complains Charles de La Condamine [17].

Sometimes order outsprings from chaos. That was just the case with metrology in France.

Remarkably, the American, British and French attempts of reform were originally based on the length of a pendulum that takes one second per swing. However, as it is well known, the French metric system was finally based on the size of Earth. But the decision to switch from the pendulum to the meridian was so sudden and

Ref. [10]), was crucial to reach, soon or later, some national and international agreement.

[8] It is a matter of fact that ancient standards are lost forever and the interpretation of data taken with those units can only be guessed somehow. A relative recent episode of an important set of standards lost by accident is the fire in the British Houses of Parliament in 1834 (see e.g. [11]).

hurried, especially when analyzed after two centuries, that it looks like a *coup de main*. Therefore, before describing the steps that led to the definition of the meter based on the Earth meridian, let us shortly review the several proposals of relating units of length to the period of the pendulum.

3 The seconds pendulum

The idea of basing units of length on nature had been advocated far before it reached definitive success with the advent of the French Revolution. Though there were also proposals to relate the unit of length to the size of Earth (see Section 4), the unit that came out quite *naturally*[9] — or at least this was the proposal that had most consensus — was the length of a pendulum oscillating with a given, well defined period.

This is not surprising. After the first intuitions and pioneer studies of Galileo Galilei at the end of the 16th century and the systematic experimental and theoretical researches of several scientists throughout the 17th century, the properties of the pendulum were known rather well. The practical importance of the principle of the pendulum was immediately recognized, and the first pendulum clock was realized in 1657 by Christian Huygens. In particular, it was known that the period of 'small oscillations' of a simple pendulum at a given place depends practically only on its length (see Appendix A). In other words, the pendulum was seen as an object capable to relate space to time [18, 19]. Therefore, discovering the possibility of grounding the unit of length, imperfect and arbitrary since ever, to something regular and constant, as the alternation of days and nights, must have been seen with enthusiasm by many scientists.

At that time there was little doubt about what the unit of time should be. The rotation of Earth had since ancient times provided a reference for units of time, such as seconds or hours. The latter stem from the subdivision of the day in 24 stages (12 during the daylight and 12 during the night) made first by the ancient Egyptians [6, 9], and that has its roots in the culture of the ancient Babylonians. The subdivision of hours in 60 minutes of 60 seconds had become of common use after medieval astronomers introduced it in the middle of 1200, in analogy to the ancient subdivisions of the degree in 60 minutes of 60 seconds (the name second derives from the Latin *secundus* and refers to the fact that the second is the 'second' subdivision of 'something', either the degree or the hour).

It had been experimentally observed that one of these customary subdivisions of the day — and indeed the closest to the human biological scale[10] — was obtained by a pendulum having the length in the human scale and easy to measure (about 25 or 100 centimeters, depending on whether the period or half the period was considered). Therefore it seemed quite 'natural' to use such a length as a unit. In particular, there was quite unanimous agreement in associating the second to a single swing of the pendulum, thus selecting the about 100 cm solution.[11] That pendulum was called

[9] The adjective 'natural' has been quite misused in the context of choosing the fundamental unit of length, calling natural what seems absolutely arbitrary to others, as we shall see in the sequel.

[10] It is not by chance that the smallest historical unit of time with proper name is approximately of the order of magnitude of the human heart pace.

[11] This choice is not surprising. Try to build yourself a 25 cm and a 100 cm pendula with a piece of string and a little weight, and you do not need to be an great experimenter to realize that, if you want to use one of them to define a unit length, you would prefer to work with the longer one.

Table 1: Old French units [20]. The metric conversion is fixed by the French law of 10 December 1799, that established the meter to be equal to *3 pied and 11.296 lignes*, i.e. 443.296 *lignes* (we give only the first six significant digits).

Name	*System equivalent*	*Metric equivalent*
ligne [*line*]		2.25583 mm
pouce [*inch*]	12 lignes	27.0699 mm
pied (de Roy) [(*Royal*) *foot*]	12 pouces	32.4839 cm
toise [*fathom*]	6 pieds = 864 lignes	194.904 cm
leiue postale [*postal league*]	2000 toises	3898.07 m

seconds pendulum (also *second pendulum*, or *one-second pendulum*).

The first official proposal of basing the unit of length on the pendulum was advanced by the Royal Society in 1660, after a suggestion by Huygens and Ole Rømer (based also on a study of Marin Mersenne published in Paris in 1644 [18]). The proposal was followed by an analogous suggestion by Jean Picard in 1668. A (perhaps) independent proposal was raised by Tito Livio Burattini in 1675, who called the proposed unit 'meter' and related different units in a complete system (see Appendix B).

In April 1790, one year before the work of the commission that finally decided to base a unit of length on the dimension of Earth, a project based on a unit of length determined by the seconds pendulum at the reference latitude of 45^o was presented to the National Assembly by Charles Maurice de Talleyrand [21], upon a suggestion by Antoine-Nicolas Caritat de Condorcet.

Just a few months later a *Plan for establishing uniformity in the Coinage, Weights, and Measures of the United States* [22] was presented at the other side of the Atlantic to the House of Representatives by USA Secretary of State Thomas Jefferson[12] (he became later the third president of the United States of America). Again, the unit of length was based on the regular oscillation of a pendulum, though the technical solution of an oscillating rod rather than a simple pendulum was preferred.[13]

An analogous reform of the system of weights and measures was discussed in the same years in the British Parliament. There too the seconds pendulum was proposed, obviously with London latitude as reference, advocated by Sir John Riggs Miller[14] [10, 23]. The seconds pendulum was also supported by German scientists [10].

As a matter of fact, at the time the French Academy of Sciences had to choose the unit of length, the seconds pendulum seemed the most mature candidate for the unit of length. Moreover, with some diplomatic work concerning the choice of the reference parallel — and Talleyrand was the right person for the job —, there were good chances

[12] Jefferson states to have read the Talleyrand's report to the National Academy of France when his report was practically ready. Yet the *"proposition made by the Bishop of Autun"* — this way Talleyrand was known — convinced him to change the reference latitude of the pendulum from 38^o, *"medium latitude of the United States"*, to 45^o [22].

[13] A homogeneous rod of length l oscillating from one end behaves as a simple pendulum of length $2/3\,l$. Therefore Jefferson's *second bar* was 3/2 the seconds pendulum, i.e. about 150 cm.

[14] Indeed, there were contacts between Talleyrand and Miller to collaborate towards a common solution. But due to technical and political events, the most relevant among them being certainly the French choice of the meridian, the projects based on the pendula were put aside in France, Great Britain and United States between 1790 and 1791, as we shall see later.

to reach an agreement among France, Great Britain and United States.[15]

As far as the length of the seconds pendulum is concerned, during the 18th century its value was known with sub-millimeter accuracy in several places in France and around the world, often related to work of rather famous people like Isaac Newton, Mersenne, Giovan Battista Riccioli, Picard, Jean Richer, Gabriel Mouton, Huygens, Jean Cassini, Nicolas Louis de Lacaille, Cassini de Thury and La Condamine. For example, in 1740 Lacaille and Cassini de Thury had measured the length of the seconds pendulum in Paris ($48^o\,50'$ latitude), obtaining a value of 440.5597 lignes (see conversion Table 1), corresponding to 99.383 cm. Newton himself had estimated the length of the seconds pendulum at several latitudes between 30 and 45 degrees (see Ref. [22]): his value at 45 degrees was 440.428 lignes, i.e. 99.353 cm. A measurement at the equator, made by La Condamine during the Peru expedition [24], gave 439.15 lignes (99.065 cm).

4 The Earth based units of length and the birth of the metric system

In the middle of 1790 there was quite an international convergence towards a unit of length based on the pendulum. Nevertheless, at the beginning of spring of the following year a different unit of length was chosen by the French Academy of Sciences, thus signing the end of the seconds pendulum as length standard and causing a setback of the international cooperation on units of measurements.

According to revolution style, the pace was very rapid (see central frame of Table 2). In August 1790 the French National Assembly entrusted the reform to the Academy of Sciences. The Academy nominated a preliminary commission,[16] which adopted a decimal scale[17] for all measures, weights and coins. The commission presented its report on 27 October 1790. A second commission[18] was charged with choosing the unit of length. The commission was set up on 16 February 1791 and reported to the Academy of Sciences on 19 March 1791. On 26 March 1791 the National Assembly accepted the Academy's proposals of the decimal system and of a quarter of the meridian as the basis for the new system and the adoption of the consequent immediate unit.

The guiding ideas of the French scientists are well expressed in the introduction to the document presented to the Academy:

> *The idea to refer all measures to a unit of length taken from nature has appeared to the mathematicians since they learned the existence of such a unit as well as the possibility to establish it: they realized it was the only way to exclude any arbitrariness from the system of measures and to be sure to preserve it unchanged for ever, without any event, except a revolution in the world order, could cast some doubts in it; they felt that such a system did not belong to a single nation and no*

[15] Jefferson had already accepted the French proposal of the 45th parallel, because *"middle term between the equator and both poles, and a term which consequently might unite the nations of both hemispheres, appeared to me well chosen, and so just that I did not hesitate a moment to prefer it to that of* 38^0 *"* [22].

[16] The Commission was made up of Jean Charles Borda, Condorcet, Joseph Louis Lagrange, Pierre Simon de Laplace and Mathieu Tillet.

[17] Historians generally agree that Mouton attempted the first metric system in 1670 when he proposed that all distances should be measured by means of a decimal system of units [25].

[18] This commission was made up of Borda, Lagrange, Laplace, Gaspar Monge and Condorcet. They worked in close contact with Antoine Laurent Lavoisier [26].

Table 2: Chronology of units of length based on nature: from the seconds pendulum to (a fraction of) the light second.

1644	Mersenne writes about basing the unit of length upon a pendulum length.
1660	The length of the *seconds pendulum* is proposed for unit of length by the Royal Society, upon a suggestion of Huygens and Rømer.
1668	Picard proposes the *universal foot*, equal to 1/3 of the length of the seconds pendulum.
1670	Mouton proposes a unit of length equal to *one minute of Earth's arc* along a meridian (equal to present *nautical mile*).
1675	The *catholic* (i.e. universal) *meter*, equal to the length of a seconds pendulum, is proposed by Burattini, together with a a complete system of inter-related units based on the second.
1720	J. Cassini proposes the *radius of Earth* as unit of length (whose practical unit would be its ten millionth part).
1774	La Condamine proposes a unit of length equal to the length of the seconds pendulum at the equator (439.15 lignes).
1790, April	Talleyrand proposes to the National Assembly a unit of length equal to the seconds pendulum at 45° latitude (440.4 lignes).
1790, July	Jefferson proposes to the U.S.A. House of Representative a unit of length equal to a *second rod* at 45° latitude.
1790, August	Talleyrand's proposal is recommended to king Louis XVI by the Committee on Agriculture and Commerce. As a result, the Academy of Science is entrusted of the reform of measures and weights.
1790, October 27	A first commission recommends a *decimal scale* for all measures, weights and coins.
1791, March 19	A second commission, set up on February 16, chooses the *quarter of meridian* as natural unit of length, and its 1/10 000 000 as practical unit.
1791, March 26	The National Assembly accepts the recommendations of the commissions.
1791, April	The Academy of Sciences initially confides the new measurement of the meridian to Méchain, Legendre and J.-D. Cassini (the latter two resigned).
1792, June	Led by Delambre and Méchain, the *meridian expedition* finally *begins*
1793, May 29	A third commission provides a provisional length of the *meter* as 443.44 lignes (based on the 1740 measurements by Lacaille and Cassini de Thury).
1793, August 1	The provisional meter is adopted by decree.
1795, April 7	The *Decimal Metric System* (also including including units of weight, surface and volume — but not the unit of time) is established by law.
1798,	The meridian expedition ends.
1799, March	An international commission fixes the *length of the meter* in 443.296 lignes.
1799, June 22	The meter standard, a platinum bar, is officially presented in Paris.
1812	A hybrid system of measurements is decreed by Napoleon, in which old names can be used for non-decimal multiples and sub-multiples of the meter (e.g. 1 toise for '2 meter', etc.).
[1832	The *second* is adopted in the Metric System, breaking the decimal scheme.]
1837	The 1812 decree is repealed, banning old names in France.
1840	Non metric system units become penal offence in France.
1889	The meter is redefined as the length of the 1799 platinum bar, with no longer reference to the Earth meridian.
1960	The meter is redefined as 1 650 763.73 wavelengths of the radiation corresponding to the $2p_{10}$-$5d_5$ transition of Krypton 86.
1983	The meter is redefined as the distance traveled in vacuum by electromagnetic waves in 1/299 792 458 of a second.

country could flatter itself by seeing it adopted by all the others.

Actually, if a unit of measure which has already been in use in a country were adopted, it would be difficult to explain to the others the reasons for this preference that were able to balance that spirit of repugnance, if not philosophical at least very natural, that peoples always feel towards an imitation looking like the admission of a sort of inferiority. As a consequence, there would be as many measures as nations. (Ref. [2], pp. 1-2)

Three were the candidates considered by the commission:

- the seconds pendulum;
- a quarter of the meridian;
- a quarter of the equator.

The latter two units are based on the dimension of Earth. Indeed, Earth related units had had also quite a long history, though they were not as popular as the seconds pendulum, probably because their intrinsic difficulty to be determined.[19] Mouton had suggested in 1670 [25] the unit that we still use in navigation and call now *nautical mile*: the length of one minute of the Earth's arc along a meridian, equal to 1852 m.[20] In 1720 the astronomer Jean Cassini had proposed the radius of Earth [27], a 'natural' unit for a spherical object (he had also indicated the one ten-millionth part of the radius as the best practical unit). However, neither of these old, French proposals are mentioned in the report of the commission.[21]

The quarter of the equator was rejected, mainly because considered hard to measure[22] and somehow 'not democratic'.[23]

So, we believe we are bound to decide to assume this kind of unit of measure and also to prefer the quarter of the meridian to the quarter of the equator. The operations that are necessary to establish the latter could be carried out only in countries that are too far from ours and, as a consequence, we should have to

[19] Any person can easily determine the length of a seconds pendulum within the percent level. Surveying the Earth is a problem more difficult by orders of magnitude.

[20] The unit of speed consistent with it is the *knot*, corresponding to 1 nautical mile per hour. It is particularly suited in navigation (and hence the name). For example, a ship that sails at 30 nodes along a meridian travels one degree in latitude in two hours.

[21] The Mouton's minute of meridian is just one of the possible subdivision of the meridian, namely the 324000-th part of the quarter of the meridian: we understand that the subdivision of the right angle in 90 degrees and each degree in 60 minutes and 60 seconds was judged 'unnatural' by the académiciens because not decimal. We might guess that the radius of Earth was not considered for the 'obvious difficulty' to make an *immediate* measurement from the center of Earth to its surface. However, it should be similarly obvious that it was also impossible to measure all other quantities (diameter, meridian, equator) in an immediate way. We shall come back to this point in section 7.

[22] The difficulty in measuring arcs of the equator is not only related to perform measurements in central Africa or South America. It would have required precise measurements of differences in longitude along the equator, and measurements of longitude are intrinsically much more difficult than measurements of latitude, because the former rely on absolute synchronizations of clocks in different places, and that was not an easy task at that time. (For a novelized account of those difficulties, see Dava Sobel's *Longitude* [28]. A classical novel in which practical ways to measure latitude and longitude are well described is Jules Vernes's *Mysterious Island*.)

[23] Several of the claims and the slogans of the *académiciens* show a certain degree of naïveness (frankly a bit too much for such extraordinary clever people they were: the suspicion that they had hidden purposes in mind is almost unavoidable). We shall come back to this point in section 7.

undertake expenditures as well as to overcome difficulties that would be superior to the advantages that seem to be promised. The inspections, in case somebody would like to carry them out, would be more difficult to be accomplished by any nation, at least until the progress of the civilization reaches the peoples living by the equator, a time that still seems to be unfortunately far away. The regularity of this circle is not more assured than the similarity or regularity of the meridians. The size of the celestial arc, that corresponds to the space that would be measured, is less susceptible to be determined with precision; finally it is possible to state that all peoples belong to one of Earth's meridians, while only a group of peoples live along the equator. (Ref [2], pp. 4-5)

The pendulum was rejected after a long discussion (two pages over a total of eleven of the document — the quarter of equator is instead ruled out in less than half a page). As a matter of fact, the commission acknowledges that

the length of the pendulum has appeared in general to deserve preference; it has the advantage of being the easiest to be determined, and as a consequence to be verified. (Ref [2], p. 2)

Then the report specifies that the pendulum should be a simple pendulum at the reference latitude of 45^o, because *"the law followed by the lengths of simple pendula oscillating with the same time between the equator and the poles is such that the length of the pendulum at the forty-fifth parallel is precisely the mean values of all these lengths".*[24] (Ref. [2], p. 3)

There was still the problem of the reference time of the pendulum, since the second was considered an "*arbitrary subdivision of this natural unit* [the day]" [2]. But a possible way out was envisaged:

In reality, we could avoid the last inconvenience taking as unit the hypothetical pendulum that made just one oscillation in one day; its length, divided by ten billion parts, would give a practical unit of measurement of about twenty seven pouces [27 pouces $\approx$ 73 cm]*; and this unit would correspond to the pendulum that makes one hundred thousand oscillations in one day.*[25] (Ref. [2], p. 4)

Essentially, the report of the commission does not provide any specific weakness of the pendulum[26] and, finally, the choice of the quarter of the meridian is justified only in terms of 'naturalness', as it was perceived by Borda and colleagues:

[24] Actually, such a length is not 'precisely' the average over the lengths at all parallels, but only a very good approximation. In fact, the net gravitation acceleration g does not vary linearly with the latitude, but follows the following law [24]:

$$g/(\mathrm{m/s^2}) = 9.7803185(1 + 0.005258895\,\sin^2\phi - 0.000023462\,\sin^4\phi)\,,$$

where ϕ is the latitude (the theoretical formula is valid at the ellipsoid surface and, as is customary in geodesy, g is the sum of the effects of gravitation and centrifugal forces).

[25] Ten billion is the square of one hundred thousand (the length of the pendulum is proportional to the square of its period). Seen with modern eyes, it looks a bit bizarre that this 'hypothetical pendulum', sized almost twenty times the distance Earth-Moon, would have been natural, while the second wouldn't. (By the way, for those who like to understand all digits: 73 cm comes from rounding to the pouce a length that, directly rounded to the centimeter, would be 74 cm.)

[26] It should be noted that a unit of length based on the pendulum has intrinsic problems, like the dependence of its period on temperature, latitude and above sea level, plus other more technical issues, considered also in Jefferson's document [22]. But, in the part of the report in which the seconds pendulum is discussed and rejected as unit of length, the French commission does not seem concerned

[...] one would still have to include an heterogeneous element, time, or what is here the same thing, the intensity of the gravitational force at the Earth's surface. Now, if it is possible to have a unit of length that does not depend on any other quantity, it seems natural to prefer it.[27]
[...]
Actually, it is much more natural to refer the distance between two places to a quarter of one of the terrestrial circles than to refer it to the length of the pendulum.
[...]
The quarter of the Earth meridian would become then the real unit of length; and the ten million-th part of this length would be its practical unit. (Ref [2], pp. 4-5)

It must be stressed that, however, there was still a strong resistance from those scientists who preferred the pendulum [10, 29].

The new unit was officially called 'meter'[28] only two years later (see section 5.2), in the occasion of a report of a new commission, in which also for the first time an estimate of its length was made public by Borda, Lagrange and Monge [32, 14].

5 Establishing the length of the meter

Reading the *Rapport sur le choix d'une unité de mesure* [2] we have been surprised not to find the expected value for the new unit of length. All looks as its length was unknown and it had to be determined by the campaign of measurements outlined in the document. The seconds pendulum has the same omission. But we imagine that the members of the National Assembly, to which the document had to be finally read, were curious to know the rough length of the unit they were going to decree. Instead, the commission provides only an estimate of the size of the unit that would result from the ideal, 7.4 million kilometer long pendulum that beats the day. Therefore, it seems reasonable to believe that the length of the seconds pendulum and of the fourth-millionth part of the meridian were already known rather well, and that there was no need to specify their value. This was our first guess. Actually, the question looks a bit more subtle at a closer look: though those values were well known to scientists, the académiciens kept them 'secret' or, at least, they were reluctant to provide an official best estimate of the new unit of length to the politicians [10]. It seems to us that the reason of this reserve is closely related to the preference of the meridian over the pendulum. But let us proceed with order.

The seconds pendulum has been reviewed in section 3. Comparing Cassini's and Newton's value, we can safely take an approximated value of the seconds pendulum at 45^o of 440.4 lignes (99.35 cm). Let us now see how well the 'meter' was known at the

at all with this kind of physical questions. Only later (p. 9), when they propose the pendulum as ancillary reference of length, they specify that the pendulum should beat *"at the sea level, on vacuum and at the temperature of melting ice."* [2]

[27] Our note: to be precise, that is not right "*the same thing*." The pendulum relates space and time via the net gravitational acceleration g. Therefore the included heterogeneous elements are two: time and acceleration. A similar situation happens now, where the meter is related to the second via the speed of light.

[28] The name 'meter' comes from Greek *metron*, meaning 'measure'. The first who proposed the name meter in the context of the French Academy work is acknowledged to be the mathematician Leblond in 1790 [30, 31]; still, some historians (see e.g. Ref. [29]) maintain the idea has to be originally attributed to Borda. However, the name meter for a unit of length (that practically coincides with the French Academy meter) was proposed more than one century earlier by Burattini (see Appendix B).

time of the March 1791 report [2]. We shall then go through the recommendations of the commission for a more accurate determination of the unit of length and through the resulting meridian expedition.

5.1 Measurements of the Earth meridian before 1791

Measuring the size of Earth had been a challenging problem for ages, since it was first realized that Earth is spherical, i.e. at least by the sixth century B.C. [24]. The most famous ancient estimate is that due to Eratosthenes (276–195 B.C.), who reported a value of 250 000 stadia, i.e. about $\approx 40\,000$ kilometers, if we take 159 m per stadium.[29]

The principles of measurement of the Earth parameters, exposed at an introductory level, as well as milestones of these achievements, can be found in Ref. [24]. The basic idea is rather simple: if one is able to measure, or estimate somehow, the length of an arc of meridian (s) and its angular opening (α), the length of the meridian can be determined as $360^o \times s/\alpha$, if a circular shape for the meridian is assumed (i.e. for a spherical Earth). The angle α can be determined from astronomical observations. The measurement of s is bound to the technology of the epoch and varies from counting the number of steps in the early days to modern triangulation techniques [24]. Indeed, the rather accurate measurements between the 16th and 18th centuries provided the results in terms of s/α, expressed e.g. in toises/degree (see Table 3).

As we can see in Table 3, not exhaustive of all the efforts to pin down the Earth dimensions, there was quite a convergence on the length of the unitary meridian arc in Europe, as well as a general consistency with older measurements. For example, taking the Lacaille and Cassini de Thury result, based on the about 950 km arc of the Paris meridian[30] across all France, it is possible to calculate a value of 443.44 lignes for the new unit of measurements, assuming a spherical Earth ($57027\times 90^o/10\,000\,000\times 864 = 443.44$). Even a conservative estimate would give a value of 443.4 lignes, with an uncertainty on the last digit — a difference of three lignes (about 6 mm) with respect to the length of the second pendulum. That was definitely known to Borda and colleagues.

However, apart from experimental errors in the determination of the 57027 toise/degree, the value of 443.44 was still affected by uncertainties due to the shape of Earth. At that time the scientists were rather confident on an elliptical shape of the meridians, resulting from the Earth flattening at the poles. In fact, centrifugal acceleration due to rotation is responsible for the bulge of the Earth at the equator. The resulting Earth shape is such that the total force (gravitational plus centrifugal) acting on a body at the Earth surface is always orthogonal to the 'average' surface of the Earth. If that were not the case, there would be tangential forces that tended to push floating masses towards the equator, as eloquently stated by Newton: "*... if our Earth were not a little higher around the equator than at the poles, the seas would subside at the poles and, by ascending in the region of the equator, would flood everything there.*" (Cited in Ref. [33].) Newton had estimated an Earth ellipticity of 1/229 [34].

[29] Due to uncertainties in the conversion factor stadium-meter, approximations and errors in evaluating distances and differences in latitude, the usually quoted value of 40 000 kilometers obtained by Eratosthenes has to be considered fortuitous, being the uncertainty on that number of the order of 10% [24]. Anyway, not bad for that time (in frontier physics a completely new measurement that provides a result with 10% uncertainty is considered a good achievement).

[30] The Paris meridian, now $2^0 20' 14''$ East, had been the oldest zero longitude, until in 1884 it was replaced by the Greenwich meridian, even though France and Ireland adopted the new zero only in 1911.

Table 3: Some milestones in measuring the Earth meridian. l_m stands for the length of the meter calculated as the 40 000 000th part of the meridian (for some important cases, l_m is also given in lignes — see Table 1 for conversion). In the results expressed in the form 'xxxxx$\times 360^o$' xxxxx stands for the length of one degree meridian arc (s/α in the text). Ancient estimates have to be taken with large uncertainty (see e.g. Ref.[24]).

Author(s)	Year	Value	Unit	km value	l_m (m) {lignes}
Eratosthenes	(III B.C.)	250000	stadium[a]	$\approx$ 40000	$\approx$ 1.0
Caliph Al-Mamun	820	56 $^2/_3$	Arab mile[b]	39986	0.9997
Fernel	1525	56746×360^o	toise	39816	0.9954
Snellius	1617	55100×360^o	toise	38661	0.9665
Norwood	1635	57300×360^o	toise	40204	1.0051
Picard	1670	57060×360^o	toise	40036	1.0009
J. Cassini	1718	57097×360^o	toise	40062	1.0016
Lacaille and Cassini de Thury	1740	57027×360^o	toise	40013	1.00033 {443.44}
[*Lapland expedition*	*1736*	*57438* $\times$ *360*^{o}	toise	*40302*	*1.0075*] [c]
[*Peru expedition*	*1745*	*56748* $\times$ *360*^{o}	toise	*39817*	*0.9954*] [c]
Delambre and Méchain	**1799**	**20522960**	**toise**	**40000**	**1 {443.296}**
		[*57019* $\times$ *360*^{o}	toise	*40008*	*1.00019*] [d] {*443.38*}
present value		40009152	m	40009.152	1.000229 {*443.3975*}

a) Stadium estimated to be 159 m.
b) Arab mile estimated to be 1960 m [24].
c) Values obtained at extreme latitudes, very sensitive to Earth ellipticity.
d) The entries of this line assume a spherical model for Earth, as for older estimates. The value of 57019 toises per degree is obtained dividing the 551 584.74 toises of the meridian arc Dunkerque-Barcelona by their difference in latitude, $9^o\,40'\,25.40''$ [35] .

Several measurements had been done during the 18th century to determine the value of Earth flattening. In particular, there had been an enormous effort of the French Academy of Sciences, that supported measurements in France as well as expeditions at extreme latitudes, up to the arctic circle and down to the equator.[31]

The latter measurements were essential in order to gain sensitivity on the flattening effect. In fact, the unitary arc length s/α gives the local curvature along the meridian around the region of the measurements. As a consequence, $\rho = s/\alpha \times 360^o/2\pi$ is equal to the radius of the circle that approximates locally the meridian ellipse. As it can be easily understood from figure 1, the curvature decreases with the latitude: the radius of the 'local circle' is minimum at the equator and maximum at the pole. The measurements

[31]The Lapland expedition measured an arc of $57'$ crossing the north polar circle in northen Finland, at an average latitude of $66^o\,19'$ N. The Peru expedition measured an arc of $3^0\,7'$ at an average latitude of $1^0\,31'$ S (see [36] for a nice web site dedicated to the expeditions).

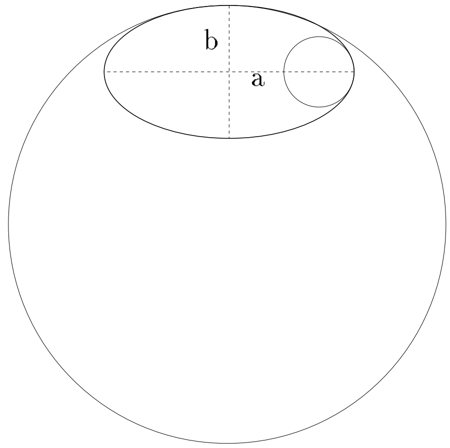

Figure 1: An exaggerated representation of the ellipsoidal Earth shape, showing local circles at the equator and at the pole (the ellipse is characterized by the semi-axes a and b). The ellipse of the figure has a flattening of about 1/2, i.e. an eccentricity of 0.87. (A flattening of 1/298, corresponding to the Earth one, i.e. a minor axis being 0.3% smaller than the major axis, is imperceptible to the human eye.) Note how the equatorial local circle underestimates the ellipse circumference, while the polar one overestimates it.

of arcs of meridian at several latitudes (two distant latitudes are in principle sufficient) can yield the ellipse parameters. Comparing the result of Lacaille-Cassini from Table 3 with the results of the Lapland and the Peru expeditions from the same table, we see that s/α is indeed increasing with the latitude (note these measurements were quite accurate — for a very interesting account of the Peru expedition see Ref. [37]). The combination of these and other measurements gave values of the Earth flattening in the range 1/280–1/310 [37], with a best estimate around 1/300, very close to the present value of 1/298 (see Table 4). However, given such a tiny value of the flattening (imagine a soccer ball squeezed by 0.7 mm), its effect on the circumference of the ellipse is very small, of the order of a few parts in 10 000.

To summarize this subsection, we can safely state that the length of the meridian, and hence of any of its subdivisions, was known with a relatively high accuracy decades before the report that recommended the unit of length equal to 1/10 000 000 part of the quarter of meridian was produced. In particular, for what this paper is concerned, it was well known that the new standard was equal to the length of the one seconds pendulum within about half percent.

5.2 The 1793 provisional meter

The first public figures for the new standard, together with the name *meter*, were provided by the académiciens in a report by Borda, Lagrange and Monge in spring 1793 [32]. A project of decree for a general system of weights and measures, also containing the cited report, was presented to the National Assembly in July of the same year [14]. The length of the meter was obtained from the Lacaille-Cassini measurements, with the simple calculation shown in the previous subsection.

> *Its approximated value is 3 pieds 11 lignes 44/100 present Paris measure* [443.44 lignes]*, and this approximation is such that its error does not exceed one tenth of*

Table 4: Earth data [38]. The geometrical data refer to the WGS84 ellipsoid [37]. Note that the generic 'radius of Earth' R refers usually to the equatorial radius, but sometimes also to the 'average' equivolume radius. In literature the name 'ellipticity' is often associated to what is called 'geometric flattening' in this table, and even to the difference of equatorial and polar radii divided by their average, i.e. $(a-b)/((a+b)/2)$. Anyway, the three different 'ellipticities' give with good approximation the same number, about 1/298, because of the little deviation of our planet from a perfect sphere. Note also that sometimes the flattening is even confused with the ellipse eccentricity, that differs quite a lot from flattening and ellipticity.

equatorial radius, a	6 378 137 m
Polar radius, b	6 356 752 m
Equivolume sphere radius	6 371 000 m
Geometric flattening, $f=(a-b)/a$	1/298.26
Ellipticity, $(a^2-b^2)/(a^2+b^2)$	1/297.75
Eccentricity, $e=\sqrt{1-b^2/a^2}$	$0.08182 = 1/12.2$
(for $f \ll 1$, e is about $\sqrt{2f}$)	
Mass, M	5.97369×10^{24} kg
Mean density	5.5148×10^3 kg m^{-3}
Normal gravity at equator	9.7803267 m s^{-2}
Normal gravity at poles	9.832186 m s^{-2}
GM (where G is the gravitational constant)	3.986005×10^{14} m^3 s^{-2}

> *ligne* [0.2 mm]*, that is sufficient for the ordinary use in the society. This unit will take the name meter.* ([14], p. 5)

The so called *provisional meter*, with all the resulting units of volume, weights and surfaces [14, 32], was adopted by decree the 1st August 1793.

5.3 The 1792-1798 meridian mission

The *Rapport sur le choix d'une unité de mesure* [2] outlined a plan to realize the new system of measures. One of the items of the plan was to re-measure the Lacaille-Cassini arc of the Paris meridian from Dunkerque to Perpignan,[32] also extending the measurements down to Barcelona (a total of almost ten degrees in longitude, for a length of 1075 km — see the map in Ref. [39] to get an idea of "*cette belle entreprise*"[14]).

The motivation of the Academy in support of the enterprise was to improve the knowledge of the meridian length. The confidence on a possible improvement relied on several factors:

> *the ability of the astronomers presently involved in this job, the perfection that the mathematical tools and instruments have acquired in the last times, the magnitude of the measured circle, that is extended by more than nine degrees and one half, the advantage of being this* [the meridian arc] *cut in the middle by the forty fifth parallel, all that guarantees us the exact and perfect execution of this beautiful enterprise, the greatest of this kind. (Ref. [14],* pp. 3-4)

[32] More precisely, the lowest latitude was about Collioure, a small town close to the Spanish border.

The campaign of measurements started immediately, apart from delays of practical nature, after the Academy's proposal was accepted. A quote is in order to report the driving spirit of the commission, that also shows the yearning for an 'egalité of measures', one of the political and ideological requirements of the new régime that can't be set aside any longer.

> *There is no need, in our judgment, to await the concurrence of other nations either to choose a unit of measurement or to begin our operations. Indeed, we have eliminated all arbitrariness from this determination and rely only on information equally accessible to all nations.* (Ref. [2], p. 11)

Two astronomers were nominated responsible of the mission: Jean Baptiste Joseph Delambre, in charge of the northern part of the arc, up to Dunkerque, a French town on the North Sea, close to the Belgian border; Pierre François André Méchain, in charge of the southern part.

In principle, the task was simpler than that accomplished by Lacaille and Cassini fifty years earlier, because it seemed initially possible to use much of their work (like the triangulation stations). However, things were much more difficult, complicated by Revolution and wars (see Refs. [40, 10] for a dramatic account of the enterprise). It was almost a miracle that Delambre and Méchain could meet again in Paris in November 1798 alive and with the logbooks of their measurements.

The unexpected long duration of the mission was the reason the académiciens were urged to provide the provisional length of the meter in 1793. In fact, "*the interests of the Republic and of the commerce, the operations initiated on the money and on the cadastre of France, require that the adoption of the new system of weight and measures is not delayed any longer*" [14]. The *Decimal Metric System* was later established by law on April 7, 1795, well before the meridian mission was accomplished.

After the end of the meridian mission an international commission was convened to review the Delambre-Méchain data and to establish the length of the meter. In March 1799 the meter was determined in 443.296 lignes, also taking into account Earth flattening.[33] The new standard differed by 0.114 lignes (0.32 mm) from the provisional unit. Compared with our present value (see Table 3) one can see that the new result slightly worsened the knowledge of the meridian.[34]

[33] The Dunkerque-Barcelona arc is sufficiently large to allow an estimate of the meridian curvatures in several sub-arcs and to make an independent estimate of Earth flattening. That came out to be about one half of that based on many data sets from equator to Lapland. The flattening based on the latter information was finally preferred, As Alder puts it "*. . . 1/150 offered the best description of the arc as it passed through France, but they knew that the older data offered a more plausible picture of the overall curve of the Earth. They could choose consistency or plausibility. And after some heated discussion, they chose plausibility and the old data.*" Actually, the discrepancy between the values of flattening in different sub-arcs was a first indication that Earth has a more complicate shape than just a rotational ellipsoid, giving rise to the concept of *Geoid* (see e.g. [24]).

[34] We would like to point out that the two results would come to a better agreement if they were treated in the same way. In fact, correction for flattening was not applied to the Lacaille-Cassini result. In Table 3 we have done the exercise of calculating the length of the meridian from s/α of Delambre-Méchain, equal to 57019 toises/degree. We obtain a resulting meter of 443.379 lines. Since the Lacaille-Cassini arc was roughly similar to the Delambre-Méchain one, we can use the ratio 443.296/443.379 as an approximate correction factor to take into account Earth flattening in the Lacaille-Cassini data. After the correction the meridian length becomes 40006 km and the corrected provisional meter would be 443.36 lignes (and $l_m = 1.00014\,\mathrm{m}$), i.e. a difference of only 0.064 lignes (1.4 mm) with respect to the 1799 final meter. Anyway, though the two results get closer, the one based on the Lacaille-Cassini measurements gets also slightly closer to the present value of the meridian length.

The manufacture of the definitive model, based on the results on those measurements, was completed in June 1799. On 22 June, the prototype of the meter was solemnly presented to the Council of Elders and of the Five Hundred.

6 Interplay of pendulum and the meridian based standard

As we have remarked several times, officially it seems that the meter was invented 'out of nothing', apparently only stemming from the slogan "*ten million meters from the pole to the equator*" [41]. A pendulum had been considered as a possible candidate as unit of length, but it had been rejected to avoid a unit of length depending on a unit of time. However, in listing the necessary operations in order to realize the reform of measures, a *unitary pendulum* is taken into account, as a kind of secondary standard to reproduce the meter:

> *The operations that are necessary to carry out this work are the following: ...*
> *4th To make some observations in latitude forty-fifth degree to verify the number of oscillations that a simple pendulum, which corresponds to the ten millionth part of the arc of a meridian, would accomplish within a day, in the vacuum, at sea level, at freezing point, so that, after having learned that number, this measure could be found again through the observations of the pendulum. In this way the advantages of the system that we have chosen are joined to the advantages that would be obtained by taking the length of the pendulum as a unit. It is possible to accomplish these observations before learning this ten millionth part. Actually, if the number of the oscillations of a pendulum having a determined length are known, it will be sufficient to learn the relation between this length and the ten millionth part to infer undoubtedly the investigated number.*[35] (Ref. [2], p. 9)

Therefore, while in the first part of the report the one-second pendulum was ruled out, a one-meter pendulum[36] appears in the second part.

The fact that the meter pendulum had to be considered an important tool to carry out the entire project can be easily inferred from the response of the President of the National Assembly to a memory of Borda [42] on the metric system of weights and measures in 1792:

> *You took your theory from the nature: among all the determined lengths you chose the two unique lengths whose combined result was the most absolute, the measure of the pendulum and above all the measure of the meridian; and by so relating the first to the second with as much zeal as sagacity, the double comparison of time and Earth through a mutual confirmation, you will have the honor to have discovered this permanent unit for all the world, this beneficent truth that is going to become a new advantage to the nations and one of the most useful conquest of the equality.* (Ref. [42], p. 9)

And, again, in the July 1793 proposal of decree, it is stated that

> *the academy has judged that its works were quite well advanced, and that the arc of meridian, as well as the length of the seconds pendulum, the weights of*

[35] The task specified in the 4th point was later committed to Borda and Charles Augustin de Coulomb.

[36] This name is not appropriate for the pendulum mentioned in the 1791 document, as the name 'meter' had still to be made official. Nevertheless, let us call it so hereafter.

a cubic pied of distilled water, were known at this moment, both from previous observations as well as from those on which the members of the commission have been working, with the accuracy sufficient to the ordinary usages of the society and the commerce (Ref. [14], p. 4)

Therefore, while in the first quote of this section (Ref. [2], p. 9) the commission generically speaks of a "*a pendulum of a determined length*", it is here clear that the referred pendulum was the seconds pendulum, and not a pendulum based on a previous unit of length, like a one-toise pendulum. We see clearly the advantage of that choice. Since the seconds pendulum was quite well known, including the dependence of its period with latitude, once the oscillation time of the meter t_m was known, the unitary length would have been easily determined as $u_m = l_{sp}\, t_m^2$, where l_{sp} indicates the length of the seconds pendulum and t_m is expressed in seconds. However, it was never stated that the seconds pendulum and one-meter pendulum are very close in length, though their closeness makes easier the intercalibration procedure.

7 Conclusions and discussion

The initial aim of this paper was to rise the question that gives the title, why the semi-period of a meter simple pendulum is approximately equal to one second. In fact, it seems to us that this question has never been raised in literature, not even at a level of a curiosity (for some references on the subject, see e.g. [29, 35, 40, 10, 43]).

We were fully aware that mere coincidences are possible and that speculations based on them can easily drift to the non-scientific domain of numerology, especially in the field of units of measures, where the large number of units through the ages and around the world cover almost with continuity the range of lengths in the human scale. Therefore, we were searching if there were reasonable explanations for the close coincidence pointed out in our question.

A physical reason is ruled out. A unit of length defined as a fraction of a planet meridian makes the period of a unitary pendulum only depending on the planet density. But this period has no connection with the planet rotation period, and then with its 86400th part.

The suspicion remains — definitely strengthened — that, among the possible choices of Earth related units, the choice favored the one that approximated a pendulum that beats the second.

7.1 About the choice of the meridian

As we have remarked in Section 4, the *Rapport sur le choix d'une unité de mesure* [2] shows a certain degree of naïveness. For example, let us take the preference of the meridian over the equator, justified in terms of ease of measurement and of universality with respect to all nations of Earth.

Let us start from the latter point, that we called 'democratic'. The sentence "*it is possible to state that every people belongs to one of the Earth's meridian, while only a group of people live along the equator*" is not more than a slogan. It is self-evident that every point on the Earth surface belongs to a meridian. A different question is to measure it in order to reproduce the meter with the accuracy required to exchange the results of precise measurements in different places of the globe. Even assuming that every people had the proper technology to perform the measurements, the country should be extended enough along the longitude in order the required measurements to be performed. Moreover, since Earth is flattened at the poles, at least two measurements of arc of meridian at two distant latitudes are needed, in order to infer the Earth ellipticity. Therefore, also the sentence "*The operations that are necessary to establish the latter could be carried out only in countries that are too far from ours*", referred to the equator, applies also to the meridian, with even a complication for the latter: while the determination of the equator requires only one campaign of triangulation, because (apart from small mass dishomogeneity) the circularity of the equator comes from symmetry arguments, determining the meridian requires necessarily, as it had already been done in the middle of the 18th century, several campaigns at different latitudes.

Frankly, we find that naïveness acquires an unintentional humorous vein at page 8 of the document [2], when, after having claimed a few pages earlier that the choice of the meridian is 'democratic', the Paris meridian going from Dunkerque to Barcelona is presented as almost unique to perform the proposed measurement:[37]

> *One cannot find neither in Europe nor in any other part of the world, unless to measure a much wider angle, a portion of meridian that satisfies at the same time the condition to have the extreme points at sea level, and that of crossing the forty-fifth parallel, if one does not take the line that we propose, or as well another more western meridian from the French coast, until the Spanish one. (Ref. [2], p. 8)*

(Their precise Swiss neighbors would have no chance to reproduce the meter standard in their country!) As a matter of fact, the choice of the meridian appeared to other countries, especially Great Britain and USA, as a imposition of the Paris meridian. All previous attempts of cooperation towards an international standard of length were frustrated, and still now we suffer of communication problems.[38]

Let us come now to the concept of 'naturalness', about which we have already expressed some *caveat* above. Once chosen Earth as reference object upon which the scale of lengths has to be based, which of its parameters is the most natural? For simplicity, let us consider a sphere. To a mathematician or a physicist the natural parameter of the sphere is the radius (that was basically the reason of Cassini's proposal mentioned in section 4). However, for a engineer the natural parameter is the diameter, because that is what he directly measures with a gauge in the workshop. The diameter is also the convenient parameter for a sphere seen from very far (as it could be a planet). But if we take a soccer ball, neither of the above two parameters is 'natural'. It is not by chance that the FIFA laws establish the ball size by its circumference ["*of not more than 70 cm (28 inches) and not less than 68 cm (27 inches)*" [45]], for it can be easily checked with a tape-measure.

[37] That could be the reason why the world best selling encyclopedia erroneously reports that the meter "*was originally defined as one ten-millionth of the distance from the equator to the North Pole on a line running through Paris.*"[44] This could be just a minor flaw due to superficiality, but it could also be a heritage of the anglo-saxon reaction to what was perceived as a French imposition.

[38] As a side remark, we would like to point out that even the very interesting Alder's book *The Measure of All Things* [10], that has been very useful to us in this research, is a proof the dreams of the académiciens are still far from coming true. In fact most lengths are given in feet and inches, used by to American and British readers, but hard for the others, especially when, in translation, unavoidable mistakes happen, as the 25 feet of page 188, that becomes 672 meters at page 289 of the Italian edition. Nothing compared to the Mars Climate Orbiter disaster, but this is symptomatic of the troubles that disuniformity of units of measures still causes, made worse by globalization.

The situation is a bit more complicated for a sphere as large as the Earth, and on which we live. It is a matter of fact that at that time it was practically impossible to make immediate measurements of *any* of the lengths related to the dimensions of Earth. One could only perform local measurements and extend the results to the quantity of interest, assuming a geometrical model of Earth. However, once a geometrical model is defined, it becomes of practical irrelevance which parameter is considered as unit, that could be the meridian, the equator or the distance between poles and center of Earth.[39]

7.2 About the choice of the quarter of the meridian

It is clear that, once the académiciens were bound to the decimal system, decided by the first commission, and the unit of length had to be related to Earth dimensions, the unit of length of practical use had to be a small decimal sub-multiple of *an* Earth dimension. But why the quarter of the meridian, instead of the meridian itself? The *Rapport* [2] does not give any justification of the choice, as if all other possibilities were out of question. And this is a bit strange. The meridian as unit of length had no tradition at all, and there had been no discussion about which submultiple to use. Evidence against the naturalness of the quarter of meridian seems to us provided by the fact that the vulgarization of the definition of the meter, as it is often taught at school and as it is memorized by most people, is the forty millionth part of 'something', where this 'something' is often remembered as the 'equator' or the 'maximum circle'.

It could be that we have nowadays a different sensitivity to the subject (we have made a little poll among friends and colleagues, and our impression has been unanimously confirmed), but we find it hard to be rationally convinced by the arguments of the following kind:

> *Once it has been chosen as base, will either the whole meridian or a sensible part of it be taken as a unit? The wholeness? Out of question! The half, that stretches from one pole to the other, may not be easily conceived by our mind because of the part which is located "below", in the other hemisphere. This is not the case of the quarter of the meridian that, on the contrary, can be easily imagined: it stretches from "one pole to the equator". In the future it will be said: France opened the divider and pointed it on one pole and the equator, a sentence that will be greatly successful. There is another reason, that is really scientific and supports the meridian: its quarter is the arc intersected by the right angle. That's right: however, why should it be considered as a further advantage? Simply because the right angle is considered as the natural angle, the angle of the vertical and the*

[39]This remains true also if an elliptic, rather than spherical model, is considered, though two parameters have to be taken into account, instead then just one. One might argue that the meridian has the advantage that it is bound only to the assumed rotational symmetry of Earth, and not to its particular shape (sphere, ellipse, or even something more complicate). But, apart from the fact the second commission report [2] speaks explicitly of measuring an arc of meridian, this possibility would imply to envisage a campaign of triangulation from the pole to the equator, that would have been infeasible at that time. (Remember that before 1909 the north pole was just an hypothetical place never reached by human beings.) As a matter of fact, the preferred parameters of modern geodesy to characterize the Earth ellipsoid (also called 'spheroid') are the equatorial radius and the flattening. In particular, the latter is the best determined Earth parameter, given with 12 significant digits (the value of table 4 has been rounded): $1/f = 298.257\,223\,563$ (WGS84) [24, 46].

Table 5: Some possible choices of units of length based on the dimensions of Earth, assumed to be spherical, together with a reasonable decimal sub-multiple as practical unit and the half period of the simple pendulum of such practical unit. (Analogous quantities can be defined assuming an ellipsoid).

unit	decimal sub-multiple	practical unit (cm)	$T/2$ (s)
radius	1/10 000 000	64	0.803
diameter	1/10 000 000	128	1.135
meridian	1/100 000 000	40	0.635
1/2 meridian (pole-pole)	1/10 000 000	200	1.419
1/4 meridian (pole-equator)	1/10 000 000	100	1.004
45th parallel	1/100 000 000	28	0.534
one radiant along the meridian (same as radius)	1/10 000 000	64	0.803
1 degree of Earth's arc	1/100000	111	1.057
1 minute of Earth's arc$^{(*)}$	1/1000	185	1.367
1 second of Earth's arc	1/100	31	0.558

$^{(*)}$ Equal to 1 nautical mile, that is 1852 m.

gravity. It is the unit-angle, the degree is nothing but its subdivision. (Ref. [35], p. 55 of the Italian translation)

What would be the alternatives? As an exercise, we show in table 5 some possible 'natural' choices of units of length based on the dimensions of Earth, together with a reasonable decimal sub-multiple as practical unit. Sub-multiples of the length of the meridian, e.g. one part over 10 000 000 or one part over 100 000 000, had led to a 'meter' of 400 or 40 of 'our' centimeters. The former is certainly too large, but the latter is quite appropriate for daily use, and, indeed, it falls in a range of length that is better perceived by people (one of the criticisms to the meter is its unnaturality, at least compared for example to the foot or even to the first standardized unit, the cubit). Even the pole-to-pole arc would have yield a better practical unit, very close to the toise.

We see from table 5 that the 10 000 000 part of the quarter of meridian is the closest to the length of the seconds pendulum. So, when the French scientists proposed the new unit of length, we think it is possible, among the many 'defensible natural units', they chose the closest to the seconds pendulum. The reason could be a compromise with the strenuous defenders of the seconds pendulum. Or it could have happened that, since they had in mind some 'cooperation' between the new unit and "*a pendulum having a determined length*" [2], choosing a unit close to the well studied seconds pendulum would have simplified the intercalibrations.

7.3 About the naturalness of a system of units

The main reason to reject the seconds pendulum was "*to have a unit of length that does not depend on any other quantity*" [2]. Now we think exactly the other way around, and prefer a system with a minimal number of units connected by physical laws, as it was suggested first by Burattini in 1675 (see Appendix B). Besides cultural aspects, that make change 'what is perceived as natural' with time,[40] we find a certain contradiction in the use of the naturalness concept expressed in the *Rapport* [2]. Why not to extend it also to the weight unit, instead of binding, as it is known, this unit to the unit of length and to the density of water?

A similar comment applies to the right angle as the "*natural angle*" to justify the quarter of meridian (see Guedj's quote in subsection 7.2): the right angle is certainly the natural one for a square or a rectangle, but why should it be natural for a circle, where there are no angles? (At most, if there were an angle to be considered natural, that would be the radiant, as all those who use trigonometric functions of computer scientific libraries know).

7.4 About the reticence concerning the value of the new unit of length

As we have stated in section 5, we have been surprised not to find an estimate of the length of the unit proposed in the second commission report [2]. Perhaps they were really afraid that, if the members of the National Assembly had known that length was determined within a few parts in ten thousand, then the new meridian mission would have not been financed [10]. In fact, when two years later they had to finally release an official number, some académiciens saw in the provisional meter the end of the meridian mission.[41]

There is another point, contained in the documents that led to the provisional meter [32, 14], that has puzzled us. Why did they not provide their 'best value', that included the best understanding of Earth flattening? As we have seen in footnote 34, the flattening correction to the meter is about two parts in ten thousand. If applied, it would have changed the provisional meter from 443.44 lignes to about 443.36 lignes. A little difference, but still relevant, if compared with the significant digits with which the provisional meter was given. We are not arguing on the base of after-wit arguments (the present best value of the 1/10 000 000 of the quarter of meridian is 443.3975 toises, right in the middle between the two values). Our question is only why they did not report the 'best value' — and they knew that 443.44 lignes was not the best value deriving from their status of knowledge, because that value assumed a spherical Earth, an hypothesis already ruled out by theoretical considerations and experimental results.

The simplest reason could be they considered that value good enough for practical applications and, since the provisional meter had to be most likely revised, it was not worth applying the little correction that was of the same size of the expected error. But then, why to state that "*its error does not exceed one tenth of ligne*", if the error due to the omitted correction counted already by about 0.08 lignes? (We understand that statements concerning errors have always to be taken *cum grano salis*, but the expression "*does not exceed*" is quite committing). Frankly, we do not see any plausible and consistent explanation, other than the somehow malicious guess, to be taken with

[40] With this respect, a suggestion put forward in 1889 by Max Planck has been particularly influential. He proposed that systems of units should be based on values assigned conventionally to certain fundamental physical constants. The first (partial) realization of Planck's idea took place in 1983 when the constancy of the speed of light in different inertial frames, adopted by Albert Einstein as the grounding principle of special relativity, was finally used to relate the unit of length to the unit of time.

[41] "*The new measures are being adopted for the commerce independent of the new measure of the earth; so there is little need for you to push yourself too hard to bring your result now*", wrote Lalande to Delambre (cited in Ref. [10]).

the benefit of inventory, that they wanted to have some room to modify the provisional meter after the end of the new measurements, in the case the new result would come very close to the old one. That would have justified the expensive enterprise.

7.5 About hidden motivations of the meridian mission

The suspicion that the choice of the meridian based unit of length was just a veil for other reasons is not a new one. Many arguments are given for example in the captivating Alder's *The measure of all things*[10]. We are inclined to believe in the most noble cause advanced in this book, i.e. the académiciens were mainly interested in making a decided step in the understanding of the shape of our planet. Borda's interest to see his *repeating circle* protagonist of a great enterprise seems also a good reason, but we see it at a second level. (It seems to us the anonymous' quote "*Sometimes to serve the people one must resolve to deceive them*", reported in Ref.[10], is quite appropriate to the case.)

However, and that was the irony of fate, the improved knowledge of the Earth decreed that the basic premise on which the meridian mission started was wrong: all meridians are different from each other, and the different arcs of the same meridian are unique.[42] The definition of the meter had then an intrinsic uncertainty, such that it would have become soon or later unsuitable for a standard of length.[43]

A final reflection

We have reasoned the plausibility of an original meter bound to the second through the swing of the pendulum. If our guess is true, then, despite the search for standards taken from nature has been the motivation for so many, long scientific and philosophical researches, at the bases of the system virtually used all over the world there are the throbs of our hearts. May be an original support to the well known statement by Jean Jacques Rousseau "*Nothing is less in our power than the heart, and far from commanding we are forced to obey it.*"

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[42] Even not taking into account asperities of the ground, hills and mountains, the concept of spheroid (or solenoid), is just a first approximation of the Earth shape (the 'zero-th order' approximation is the sphere). The equipotential surface of Earth has a complicate shape called Geoid, of which the spheroid is a kind of best fitting curve (see e.g. Ref.[46] for an introduction, that also contains a visual representation of the Geoid).

[43] That is due neither to "*le difficoltà ad applicare tale definizione, causate dalla forma sferica della terra*" (*the difficulties to apply such a definition, caused by the spherical shape of the Earth*) [47], nor because "*later it was discovered that the Earth is not a perfect sphere*"[44].

Appendix A: The local 'meter' and 'second' in the planets of the solar system

The well known small angle formula that gives the period T of the simple pendulum (i.e. the elementary text book pendulum) as a function of its length l is

$$T = 2\pi\sqrt{\frac{l}{g}}, \tag{1}$$

where g is the gravitational acceleration, approximately equal to $9.80\,\mathrm{m/s^2}$ on Earth. For $l = 1$ m we get $T = 2.007$ s. Therefore, each swing takes 1.0035s, that differs from a round second only by a few parts per thousand. Varying g by $\pm 0.3\%$ (i.e. from 9.77 to $9.83\,\mathrm{m/s^2}$), the period changes only by $\pm 0.15\%$.

In order to understand if there is any physical reason behind this numerical coincidence let us try to understand the property of Earth that mainly influences the period of the pendulum, and if there is any simplification due to the fact that the length of the pendulum is about 1/40 000 000 of the meridian.[44] The gross value of g depends on mass and radius[45] R of the Earth with local effects due to not exact sphericity (see Table 4), mass dishomogeneity and above sea level height. Moreover, there is a centrifugal term, null at the pole and maximum at the equator, due to Earth rotation.[46] In the approximation of a perfect sphere, the gravitational acceleration g, i.e. the gravitational force F_G divided by the mass of the pendulum, is given by

$$g = \frac{F_G}{m} = \frac{1}{m}\frac{G\,M\,m}{R^2} = \frac{G\,M}{R^2}, \tag{2}$$

[44] One might also think of a simplification due to the fact the length of the pendulum is unitary. To readers with little physics background we would like to make clear that what matters for the pendulum period is the length, and not the unit in which the length is expressed. The value of a generic quantity Q is given by $Q = \{Q\}\cdot[Q]$, where $[Q]$ is the *unit of measurement* and $\{Q\}$ the *numerical value*, e.g. $l = 1.38$ m. If we change unit of measurement from system A to system B , $\{Q\}$ and $[Q]$ change, preserving Q invariant:

$$Q = \{Q\}_A \cdot [Q]_A = \{Q\}_B \cdot [Q]_B\,.$$

The two numerical values are then related by

$$\{Q\}_B = \{Q\}_A \cdot \frac{[Q]_A}{[Q]_B}\,.$$

If we call M a different unit of length, such that $1\,M = \alpha$ m, we get $\mathrm{m/M} = 1/\alpha$. Therefore, a length $l = n$ m will be *expressed* as $l = N\,\mathrm{M} = n/\alpha\,\mathrm{M}$ in the new unit (but the *length* is the same: you will not grow up, if your height is expressed in centimeters or millimeters rather than in meters). As far as Eq. (1) is concerned, the numerical value of g will be transformed in the same way as l, namely $g = (9.8/\alpha)\,M/s^2$. As a consequence, the conversion factor α simplifies in Eq. (1) and the period will remain the same, as it must be.
On the other hand, if we consider a *different* length, that has unitary numerical value in the new unit M, we get a *different* period of the pendulum, namely $T' = 2\pi\sqrt{1\,\mathrm{M}/(9.8/\alpha)\,\mathrm{M/s^2}} = \sqrt{\alpha}\,T$. (For example, if the new unit of length is twice the meter, the half period of a simple pendulum of unitary length will be 1.419 s).

[45] The generic 'radius of Earth' R refers usually to the equatorial radius and it implies that Earth is considered sufficiently spherical for the purpose of the calculations.

[46] At the equator, the negative centrifugal acceleration gives a contribution to g equal to $\Delta g_c = -v^2/R = -4\pi^2\,R/T_{rot}^2 = -0.034\,\mathrm{m/s^2}$ ($T_{rot} = 86400$ s stands for the rotation period). Note that, however, in geodesy 'gravitational acceleration' g indicates the overall free fall acceleration experienced by a body and takes into account the genuine gravitational force and the centrifugal one.

Table 6: Some physical data about the planets of the solar system, together with the 'planet meter' ($l_m = \pi/2 \times 10^{-7}\,R$), the half period of a 'planet meter' pendulum [$T(l_m, g)/2$] and the 'planet second' [$T_{rot}/86400$]. Note that Eqs. (2)–(4) have been evaluated assuming perfect spherical and homogeneous planets, while the 'radius' is just one half of the equatorial diameter, and the half period $T(l_m, g)/2$ is directly evaluated from nominal value of g given in this table [48]. The minus sign in the period indicates retrograde rotation.

Planet	Physical data [48]				One 'meter' pendulum and its period		
	Mass (kg)	Radius (km)	ρ (g/cm^3)	g (m/s^2)	l_m (m)	$\frac{T(l_m,g)}{2}$ (s)	$\frac{T_{rot}}{86400}$ (s)
Mercury	$3.30\,10^{23}$	2440	5.43	3.70	0.38	1.01	58.6
Venus	$4.87\,10^{24}$	6052	5.24	8.89	0.95	1.03	−243
Earth	$5.98\,10^{24}$	6378	5.52	9.80	1.00	1.00	1.00
Mars	$6.42\,10^{23}$	3397	3.93	3.69	0.53	1.19	1.03
Jupiter	$1.90\,10^{27}$	71492	1.33	23.17	11.23	2.19	0.41
Saturn	$5.68\,10^{26}$	60268	0.69	8.98	9.47	3.23	0.45
Uranus	$8.68\,10^{25}$	25559	1.32	8.71	4.01	2.13	0.72
Neptune	$1.02\,10^{26}$	24766	1.64	11.03	3.89	1.87	0.67
Pluto	$1.27\,10^{22}$	1137	2.06	0.66	0.19	1.64	−6.39

where $M = 5.98\,10^{24}\,\mathrm{kg}$ is the mass of Earth and $G = 6.67\,10^{-11}\mathrm{N}\cdot\mathrm{m}^2\cdot\mathrm{kg}^{-2}$ is the gravitational constant. Expressing the mass in terms of density ρ and volume $V = 4/3\,\pi\,R^3$, we get

$$g = \frac{4/3\,\pi\,\rho\,G\,R^3}{R^2} = \frac{4}{3}\,\pi\,\rho\,G\,R. \tag{3}$$

The gravitational acceleration g is then proportional to the planet size and density. Let us now calculate the period of a pendulum whose length is 1/40 000 000 part of a meridian of a spherical planet, i.e. $l_m = \alpha\,R$, where $\alpha = 2\pi/40\,000\,000 = \pi/2 \times 10^{-7}$ is the fixed ratio between this 'meter' and the planet radius. The period of such a 'planetary meter' pendulum is

$$T(l_m) = 2\pi\sqrt{\frac{\alpha R}{4/3\,\pi\,\rho\,G\,R}} = \frac{\pi}{\sqrt{2/3 \times 10^7\,\rho\,G}}\,, \tag{4}$$

and depends only on planet density, and not on planet mass and size separately. In particular, in the inner planets and Earth, for which the density is approximately 5.5 g/cm^3, such a 'planetary meter' pendulum would beat approximately the second (see Tab. 6).

However, the half period of this pendulum is approximately equal to the 1/86400 part of the planet rotation only for Earth and Mars, which have approximately equal 'days'. For all other planets, the local day can be very different with respect to the Earth one. In fact, the rotation speed is related to the initial angular momentum when the planet was formed and there is no reason why it should come out to be the same in different planets (Venus and Pluto are indeed retrograde, i.e. they rotate East-West).

Appendix B: Tito Livio Burattini's catholic meter

Among the several scientists that advocated the seconds pendulum as unit of length it is worth to emphasize the figure of Tito Livio Burattini, an unusual and interesting personality of the 17th century,[47] and his *Misura Universale*, published in 1675 [49]. Apart from issues of priority on the proposal of the seconds pendulum as unit of length, to which we are not interested, the historical relevance of Burattini's work resides mainly in the several modern concepts and nomenclature that appeared for the first time in his book. The most relevant of them is the idea of relating different units via physical quantities in order to set up a complete system starting from the unit of time. The sub-title in the front page of his document accounts for his ambitious proposal:

> *Treatise in which it is shown how in every Place of the World it is possible to find a UNIVERSAL MEASURE & WEIGHT having no relation with any other MEASURE and any other WEIGHT & anyway in every place they will be the same, and unchangeable and everlasting until the end of the WORLD."*[48]

Here the word *universal* is used for the first time for a unit of measurement. In the 9th page of his document (pages are unnumbered) he makes the suggestion to call *metro cattolico* (catholic meter — 'catholic' in the sense of universal) a standard realized by the pendulum:

> *So, Pendula will be the basis of my work, and from them I shall first originate my Catholic Meter, that is the universal measure, as I think I have to name it in Greek, and then I shall originate a Catholic Weight from it."*[49]

Here is finally, in the 20th page, his definition of the meter:

> *The Catholic Meter is nothing but the length of a Pendulum, whose oscillations are 3600 in a hour* [...] *as I refer to a free Pendulum, and not to those which hang from Clocks.*[50]

We think Burattini would be very pleased to learn that the unit of length of the International System differs from his meter by only half centimeter!

[47] Tito Livio Burattini (born 1617 in Agordo, Belluno, Italy and died 1681 in Krakow, Poland) was an Italian Egyptologist, inventor, architect, scientist, instrument-maker, and traveler. He was an extremely versatile person (he even designed "flying machines"!), with interests in mathematics, physics, astronomy, geodesy and economics. He spent a few years in Egypt, where he prepared a triangulation map of this country (he was also an excellent cartographer), made measurements of many pyramids and obelisks, copied monuments and tried to classify them. After some stay in Germany, he finally settled in Krakow, where he served as the King's architect. There he performed optical experiments and contributed to the discovery of irregularities on the surface of Venus, in collaboration with the astronomers Stanislaw Pudlowsky, a former student of Galileo, and Girolamo Pinocci. He became also a highly regarded maker of microscope and telescope lenses, sending some of them as gifts to Cardinal Leopold de' Medici. In 1645, he published *Bilancia Sincera*, where he proposed an improvement to the hydrostatic balance described by Galileo in his *Bilancetta*.

[48] "*Trattato nel qual si mostra come in tutti li Luoghi del Mondo si può trovare una MISURA, & un PESO UNIVERSALE senza che habbiano relazione con niun'altra MISURA, e niun altro PESO, & ad ogni modo in tutti li luoghi saranno li medesimi, e saranno inalterabili, e perpetui sin tanto che durerà il MONDO.*" (The original is for the pleasure of Italian readers.)

[49] "*Dunque li Pendoli saranno la base dell'opera mia, e da quelli cavarò prima il mio Metro Cattolico, cioè misura universale, che così mi pare di nominarla in lingua Greca, e poi da questa cavarò un Peso Cattolico.*"

[50] "*Il Metro Cattolico non è altro che la lunghezza di un Pendolo, le di cui vibrazioni siano 3600 in un hora* [...] *ch'io intendo d'un Pendolo libero, e non di quelli che sono attaccati agli Horologi.*"

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